AD-A037 318 NAVAL HEALTH RESEARCH CENTER SAN DIEGO CALIF F/G 14/2

A SET OF PDP-12 PROGRAMS FOR COMPUTING SPECTRA OF POWER, COHERE--ETC(U)

DEC 71 C H NUTE

UNCLASSIFIED 73-6 NL

1 OF 1

AD
A037318

END

DATE
FILMED
4-77

ADA037318

A SET OF PDP-12 PROGRAMS FOR COMPUTING SPECTRA OF POWER, COHERENCE, AND PHASE

Cyril H. NUTE

Dec 71

7p.

REPORT NO 73-6

MRφ11φ1

MRφ11φ1φ1

NAVAL HEALTH RESEARCH CENTER
SAN DIEGO, CALIFORNIA 92152

NAVAL MEDICAL RESEARCH AND DEVELOPMENT COMMAND
BETHESDA, MARYLAND

391 642

A SET OF PDP-12 PROGRAMS FOR COMPUTING SPECTRA OF POWER, COHERENCE, AND PHASE

Cyril H. Nute
Navy Medical Neuropsychiatric Research Unit
San Diego, California

ABSTRACT

A time-varying function may be analyzed for its frequency content by means of either the simple Fourier transformation or the smoothed power density spectrum. Two or more functions may be related through coherence and phase spectra. Working from either analog voltages or previously prepared digital tapes, a set of programs is available for making these computations. Runs have been made with artificially generated pseudo-random numbers, and the results are presented to indicate the sampling variability to be expected under certain conditions.

INTRODUCTION

Deterministic Functions

Two types of frequency analysis may be used with a sequence of digital measurements from a time-varying function. The first is classical Fourier analysis. For example, assume an epoch length of 16 seconds with a sampling rate of 128 Hz. This provides a set of 2048 raw input data points, from which a Fourier transformation will obtain 1024 sine coefficients and 1024 cosine coefficients. These may be converted to the amplitudes and phases of a set of pure sinusoids ranging from zero to 64 Hz by increments of .0625, (1/16) Hz. The basic formula states that the minimum frequency and the sharpest frequency resolution that are available are equal to the reciprocal of the epoch length. The maximum frequency that can be studied is half the sampling frequency, known as the Nyquist or folding frequency. For a deterministic waveform, like the AC voltage on a domestic power line, such an analysis might be useful for measuring the line frequency, since any other sample is likely to give results that are virtually identical. For discriminations finer than .0625 Hz, there is no alternative but to take longer samples. With a fixed computer buffer size, a trade-off must be made between the range of frequencies to be studied and the maximum frequency resolution.

Random Functions

An entirely different class of functions consists of stationary random, or stochastic processes. By definition, these do not include any pure sinusoids but may include energy that tends to lie within certain limited frequency intervals. For example, a human being with a high alpha wave will have a number of strong Fourier components in the neighborhood of 10 Hz, but no two runs will ever yield an identical set of amplitudes. To obtain meaningful (i.e. consistent and unbiased) results, one must resort to some form of statistical averaging. The squared amplitudes, proportional to either numerical variance or electrical power, of all of the frequencies within successive frequency ranges are lumped together to give relatively gross estimates of the magnitudes and locations of the dominant components. The frequencies originally obtained by simple Fourier transformation have been called the "elementary," or "line" frequencies, due to the appearance of the discrete plots of their squared amplitudes. If a number, $\underline{K}$, of these values are added together, the sum has a chi-squared distribution with $2\underline{K}$ degrees of freedom.[1] This function is the third trade-off to be considered in setting up a spectral analysis. If $\underline{T}$ is the epoch length in seconds, and $\underline{S}$ the digitizing rate in Hz, the following relations hold:

(1) Elementary frequency $= \frac{1}{T}$

(2) Maximum frequency $= \frac{S}{2}$

(3) No. of measurements $= ST$

(4) Spectral resolution $= \frac{K}{T}$

(5) Degrees of freedom $= 2K$

In brief, the analysis may be designed to maximize the confidence associated with either the height or the position of a power peak along the frequency axis, but both goals cannot be achieved simultaneously. This basic uncertainty has been shown to be mathematically equivalent to the Heisenberg principle in atomic physics.[2] Similar uncertainties apply to estimates of coherence and phase. These are derived from the complex Wishart distribution, which is an extension of the chi-squared.[3,4]

PROGRAM OPERATION

The routines described here, which have also been submitted to DECUS, are built around the Fast Fourier Transform for Real Variables, written by James Rothman and distributed under the DECUS number 8-143. Originally written for a PDP-8 without Extended Arithmetic Element, it was titled FFT-R. The version now used is FFT-E, still entirely in P-mode, but using the additional hardware and with a minor round-off error corrected in the multiplication subroutine. The coding added by the present author has been arranged to avoid the need for any changes to the Rothman package as distributed.

Two versions have been written to accept either analog voltages via the A/D converter, or digital

measurements via LINCtape. Because of the needs of our laboratory, the tape input version is the one that has been more fully tested and submitted to DECUS. The internal parameters are set up for 16 seconds of data, sampled at 128 Hz, and requiring an output frequency resolution of .5 Hz. However, these are easily converted, and the results of doing so in one instance are described below. In any case, 2048 points from one channel are stored in the upper 4K of core, while the FFT routine operates on an equal number of points from the other channel in lower core. The routine overlays the input data with its computed Fourier coefficients.

After one channel has been analyzed, the contents of the two buffers are interchanged, and the second channel is analyzed. From the two sets of Fourier coefficients, the two auto-power spectra and the real and imaginary parts of the cross-power spectrum are computed and stored in double precision integer form on an intermediate output LINCtape (Figure 1). All of these computations take about 20-25 seconds. In the analog version, intended for on-line operation, the KW12 clock is running with an interrupt routine that digitizes the voltage inputs. While a 30-second batch of data is being analyzed, a succeeding epoch is being collected and temporarily stored on LINCtape on Unit 0. As soon as the intermediate results have been written out on Unit 1, the temporary file on Unit 0 is unloaded and the cycle restarted. The process is entirely automatic, with successive groups of one, two, or three blocks written on Unit 1 with each iteration, depending on the parameters inserted to the program.

The DIAL program that does this work is filed under the name XS for the tape input version or XSPECTRM for the analog version. After a set of runs has been completed, a FOCAL-12 program, $XS,* is called in and executed for each epoch to be analyzed. Because of storage limitations on an 8K machine, overlays have to be used, with programs $XS and $TYPEXS automatically overlaying each other. They produce a final typed listing of frequency, two auto-power spectra, and coherence and phase.

*FOCAL-12 programs are distinguished by the initial $ in their names.

SAMPLE VARIABILITY

Unfortunately, spectrum analysis is not a simple tool that can be used without a good understanding of the underlying statistical principles. Although the theoretical distributions of computed sample values have been worked out on the basis of certain convenient mathematical assumptions,[1,3] one cannot be quite certain of their applicability to real problems. Besides, since coherence and phase are jointly distributed according to rules that are considerably less familiar to most users than the chi-squared distribution, special care must be exercised in their use. Some simulations have been made with Gaussian pseudo-random number sequences input as data in order to determine the sampling distributions of the computed parameters.

Figure 2 shows the program output when there is zero correlation between the two input channels. With an epoch length of 16 seconds and an output frequency resolution of .5 Hz, each of the estimated power densities, coherences, and phases has 16 degrees of freedom. Because of some shortcuts taken in setting up these runs, the total variances printed at the top of the sheet are not in the same units as the power densities given below. However, instead of being uniformly distributed, as expected, they are seen to range all the way from approximately 200 units all the way to approximately 1200 units. The coherences, which would all be zero if they were consistent and unbiased estimators of squared correlation as a function of frequency, are seen to go as high as 0.183. Now if the same data are run with a broader frequency resolution that has 64 degrees of freedom (see Figure 3), the random distribution of estimators covers a much more narrow range. The power densities vary by no more than a factor of two, and the maximum reported coherence is five times less than in the previous run.

Two more runs were made under similar conditions, except that the simulated input data sequences had a known squared cross-correlation of 0.50 (Figures 4 and 5). The distribution of the estimated power densities is about the same as before, as one might expect, but the computed coherences are much higher. In Figure 2, with only 16 degrees of freedom, the

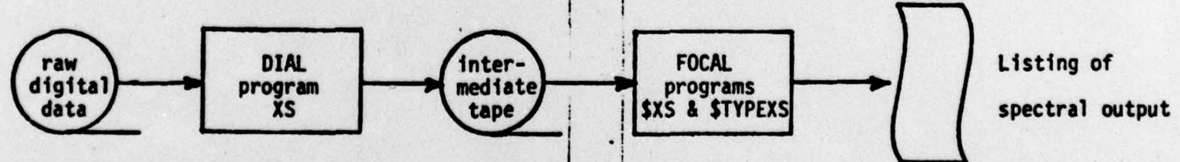

Figure 1: Steps in spectral and cross-spectral analysis

minimum coherence is less than the reported maximum where there was no correlation between the channels. It appears that with no more than 16 degrees of freedom, one cannot with complete certainty distinguish populations with zero coherence from ones with 0.50 coherence. However, there is still information to be gained from such a computer run, because in the 0.50 case, most of the computed coherences are well above the one reported in the zero case. With 64 degrees of freedom, of course, the distinction between the two populations is immediate and obvious.

CONCLUSION

A set of programs for doing spectral and cross-spectral analyses has been submitted to DECUS. They are designed for input from LINCtape and assume a sampling rate of 128 Hz over a period of 16 seconds, with an output frequency resolution of 0.50 Hz. However, other versions exist for analog input in real time with different frequency and epoch parameters. The author will be glad to advise in adapting the submitted programs to the needs of a particular user. These programs were run with simulated data and have shown that with 16 degrees of freedom, one can distinguish fairly reliably between populations whose true coherence is zero and those with a true coherence of 0.50. With 64 degrees, the distinction is much more certain, and even finer resolutions are possible.

ACKNOWLEDGMENT

The programs described here were originally commissioned by Dr. Turhan M. Itil of the Missouri Institute of Psychiatry in St. Louis, Missouri. They were written and debugged on the PDP-12 owned by the Neurosciences Department of the University of California, San Diego, Medical School, with the generous permission of Dr. Reginald G. Bickford. Subsequent development and analysis have been made with the support of the Navy Medical Neuropsychiatric Research Unit, San Diego, California, under Task MR041.01.01-0142D5CG of the Department of the Navy, Bureau of Medicine and Surgery. The opinions and assertions contained herein are the private ones of the writer and are not to be construed as official or as reflecting the views of the Navy Department.

REFERENCES

1. Blackman, R. B., and Tukey, J. W. The Measurement of Power Spectra. Dover Publications, Inc., New York, 1958.

2. Slepian, S. D., Pollak, H. O., and Landau, H. J. "Prolate Spheroidal Wave Functions, Fourier Analysis, and Uncertainty," Bell System Technical Journal XL:1, Jan. 1961.

3. Bendat, J. S., and Piersol, A. G. Measurement and Analysis of Random Data. John Wiley & Sons, Inc., New York, 1966.

4. Goodman, N. R. "On the Joint Estimation of the Spectra, Co-Spectrum and Quadrature Spectrum of a Two-Dimensional Stationary Gaussian Process," Scientific Paper No. 10, Engineering Statistics Laboratory, New York Univ., New York, 1957.

EPOCH 0
FIRST LEAD: MEAN =- 1 VARIANCE = 32959
SECOND LEAD: MEAN =- 1 VARIANCE = 34111

FREQ.	FIRST POWER	SECOND POWER	COH.	PHASE
0.50	268.02	461.20	0.551	15
1.00	521.05	592.35	0.941	1
1.50	313.24	230.03	0.166	-33
2.00	509.74	757.87	0.791	10
2.50	454.18	328.11	0.528	-7
3.00	599.28	505.98	0.202	-5
3.50	988.50	967.16	0.991	-4
4.00	452.94	502.59	0.884	6
4.50	1004.38	707.46	0.598	4
5.00	525.31	533.21	0.943	1
5.50	377.88	472.83	0.608	-11
6.00	543.04	202.99	0.241	-16
6.50	666.59	873.38	0.734	2
7.00	507.92	721.82	0.342	-7
7.50	232.48	516.11	0.371	8
8.00	704.99	615.21	0.155	37
8.50	281.67	548.39	0.154	16
9.00	731.26	564.65	0.492	-5
9.50	738.61	881.36	0.705	7
10.00	529.98	319.18	0.631	15
10.50	441.29	645.38	0.673	-4
11.00	593.16	532.90	0.677	5
11.50	760.85	551.18	0.467	7
12.00	444.34	651.38	0.819	2
12.50	1000.14	938.87	0.833	1
13.00	508.18	614.25	0.484	-2
13.50	559.58	369.00	0.504	12
14.00	741.17	917.45	0.714	10
14.50	358.79	343.31	0.283	-17
15.00	369.51	449.44	0.889	-2
15.50	192.58	489.02	0.138	-13
16.00	738.80	582.75	0.836	2
16.50	65.59	357.73	0.068	160
17.00	352.70	423.40	0.879	-1
17.50	504.20	231.17	0.207	19
18.00	595.12	695.88	0.560	-13
18.50	553.43	322.40	0.591	2
19.00	604.97	743.85	0.538	-9
19.50	859.45	1039.82	0.844	-2
20.00	604.30	688.98	0.944	-6

*

Figure 4: results of a run similar to that in Fig. 2, except that the squared correlation coefficient between the two input series = 0.5 Number of degrees of freedom = 16.

EPOCH 6
FIRST LEAD: MEAN =- 1 VARIANCE = 32959
SECOND LEAD: MEAN =- 1 VARIANCE = 34111

FREQ.	FIRST POWER	SECOND POWER	COH.	PHASE
2.00	1612.85	2041.45	0.623	5
4.00	2494.90	2303.85	0.657	-2
6.00	2450.61	1916.49	0.577	-2
8.00	2112.00	2726.52	0.355	7
10.00	2281.53	2313.58	0.481	6
12.00	2239.64	2380.84	0.625	2
14.00	2809.08	2839.56	0.653	5
16.00	1659.68	1864.52	0.526	-3
18.00	1517.60	1708.18	0.342	-3
20.00	2622.15	2795.04	0.727	-4
22.00	2466.04	2476.02	0.613	0
24.00	1931.12	2099.20	0.258	-3
26.00	2183.97	2423.75	0.488	-0
28.00	2721.80	2521.34	0.714	-0
30.00	2621.08	2589.83	0.600	-1
32.00	1681.44	1463.30	0.633	-2
34.00	1736.91	1804.63	0.599	1
36.00	2667.53	2589.68	0.645	-3
38.00	2701.70	2040.46	0.587	-1
40.00	2025.61	2394.23	0.282	9
42.00	2159.83	2274.26	0.502	2
44.00	2566.52	2197.92	0.637	-0
46.00	2491.85	2668.94	0.711	0
48.00	2116.90	2018.21	0.523	7
50.00	1578.59	1894.93	0.485	-3
52.00	2581.38	2441.25	0.673	-3
54.00	2433.84	2018.55	0.480	4
56.00	1809.71	2273.08	0.283	-5
58.00	2021.73	2051.85	0.472	6
60.00	2715.85	2310.01	0.671	1
62.00	2481.95	2423.76	0.632	-1

*

Figure 5: Results of a run similar to that in Fig. 3, except that the squared correlation coefficient between the two input series = 0.5. Number of degrees of freedom = 64.

EPOCH 1
FIRST LEAD: MEAN =- 1 VARIANCE = 34111
SECOND LEAD: MEAN =- 1 VARIANCE = 33920

FREQ.	FIRST POWER	SECOND POWER	COH.	PHASE
0.50	461.20	853.58	0.002	- 54
1.00	592.35	487.92	0.139	- 38
1.50	230.03	620.65	0.007	144
2.00	757.87	400.97	0.062	- 72
2.50	328.11	929.61	0.122	139
3.00	505.98	334.08	0.020	84
3.50	967.16	444.56	0.076	175
4.00	502.59	584.04	0.022	147
4.50	707.46	255.86	0.022	49
5.00	533.21	552.15	0.014	-145
5.50	472.83	488.52	0.053	51
6.00	202.99	720.90	0.049	76
6.50	873.38	203.86	0.150	12
7.00	721.82	552.17	0.005	170
7.50	516.11	733.49	0.036	127
8.00	615.21	600.28	0.011	-117
8.50	548.39	821.57	0.011	140
9.00	564.65	360.03	0.026	99
9.50	881.36	401.38	0.013	-180
10.00	319.18	830.83	0.037	151
10.50	645.38	370.60	0.019	-161
11.00	532.90	728.57	0.024	- 99
11.50	551.18	653.41	0.043	147
12.00	651.38	357.29	0.092	39
12.50	938.87	267.57	0.074	80
13.00	614.25	556.05	0.036	99
13.50	369.00	610.34	0.004	- 18
14.00	917.45	362.92	0.183	- 77
14.50	343.31	812.60	0.059	-163
15.00	449.44	625.45	0.023	- 55
15.50	489.02	732.08	0.050	107
16.00	582.75	629.49	0.019	160
16.50	357.73	780.22	0.062	140
17.00	423.40	552.04	0.023	84
17.50	231.17	768.48	0.032	-170
18.00	695.88	348.47	0.079	129
18.50	322.40	672.19	0.029	-132
19.00	743.85	466.37	0.050	179
19.50	1039.82	134.65	0.035	94
20.00	688.98	518.85	0.034	-158

*

Figure 2: Partial listing of the spectral analysis of two uncorrelated pseudo-random time series, with an output frequency resolution of 0.5 Hz. Number of degrees of freedom = 16.

EPOCH 7
FIRST LEAD: MEAN =- 1 VARIANCE = 34111
SECOND LEAD: MEAN =- 1 VARIANCE = 33920

FREQ.	FIRST POWER	SECOND POWER	COH.	PHASE
2.00	2041.45	2363.11	0.022	- 54
4.00	2303.85	2292.29	0.040	148
6.00	1916.49	2017.43	0.010	66
8.00	2726.52	2089.80	0.001	79
10.00	2313.58	2413.81	0.014	143
12.00	2380.84	2109.88	0.001	149
14.00	2839.56	1796.87	0.002	- 4
16.00	1864.52	2799.61	0.008	164
18.00	1708.18	2449.20	0.031	135
20.00	2795.04	1792.06	0.017	-175
22.00	2476.02	2817.27	0.010	-138
24.00	2099.20	2569.24	0.006	146
26.00	2423.75	1682.21	0.003	-119
28.00	2521.34	2102.43	0.005	- 17
30.00	2589.83	2055.56	0.005	-147
32.00	1463.30	2660.84	0.012	80
34.00	1804.63	2598.29	0.017	-103
36.00	2589.68	2194.78	0.001	107
38.00	2040.46	2008.53	0.001	83
40.00	2394.23	1718.26	0.004	-114
42.00	2274.26	2395.39	0.009	85
44.00	2197.92	1984.73	0.011	- 94
46.00	2668.94	1426.92	0.000	160
48.00	2018.21	2604.25	0.027	-129
50.00	1894.93	2641.03	0.008	174
52.00	2441.25	2182.88	0.020	117
54.00	2018.55	2451.03	0.002	-124
56.00	2273.08	2291.12	0.008	84
58.00	2051.85	2245.89	0.004	- 50
60.00	2310.01	2271.75	0.012	- 16
62.00	2423.76	1907.99	0.009	- 13

*

Figure 3: Results of a run similar to that in Fig. 2, except for the output frequency resolution of 2.0 Hz. Number of degrees of freedom = 64.

73-6

4. TITLE (and Subtitle)

(U) A Set of PDP-12 Programs for Computing Spectra of Power, Coherence and Phase

5. TYPE OF REPORT & PERIOD COVERED

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s)

Cyril H. Nute

8. CONTRACT OR GRANT NUMBER(s)

9. PERFORMING ORGANIZATION NAME AND ADDRESS

Naval Health Research Center
San Diego, CA 92152

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS

MR011.01.01-6061

11. CONTROLLING OFFICE NAME AND ADDRESS

Naval Medical Research & Development Command
Bethesda, MD 20014

12. REPORT DATE

December 1972

13. NUMBER OF PAGES

5

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)

Bureau of Medicine & Surgery
Department of the Navy
Washington, DC 20372

15. SECURITY CLASS. (of this report)

UNCLASSIFIED

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

Presentation to Digital Equipment Corporation Users Society (DECUS) Symposium, Anaheim, Calif., 30 Nov-2 Dec 1972.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Frequency analysis
power
coherence
phase

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A time-varying function may be analyzed for its frequent content by means of either the simple Fourier transformation or the smoothed power density spectrum. Two or more functions may be related through coherence and phase spectra. Working from either analog voltages or previously prepared digital tapes, a set of programs is available for making these computations. Runs have been made with artifically generated pseudo-random numbers, and the results are presented to indicate the sampling variability to be expected under certain conditions.